READJUSTMENT OF A FRONT AFTER COOLING BY PRECIPITATION

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INTRODUCTION

The problem of the readjustment of a front in unstable equilibrium, such as might be caused by sudden cooling of the cool dry air by precipitation, has been treated by Starr [1]. Two air masses differing in density by 4 percent ($\sim 10°$ C.), initially motionless, and separated by a vertical front, are allowed to come to equilibrium. For the first model, when the two air masses, 8 km. deep, comprise the entire atmosphere, the displacement southward of the front from its original position is 500 km. In the second model, the readjustment of the same two air masses takes place under a deep upper layer of 20 percent lesser density than the lower cool air, and in this more realistic case the displacement southward of the front is 450 km.

The numerical computations are extremely tedious because of the large number of significant figures which must be carried to obtain meaningful results. Hence, it does not appear worthwhile to repeat Starr's calculations for a model where perhaps only the lower 2 km. of the cool air, whose density is initially very nearly equal to that of the warm air, is suddenly cooled by precipitation 5° to 10° C. A simplified model is adopted which lends itself more readily to calculation and gives results comparable to Starr's more complicated and arduous attack.

THE FRONTAL MODEL

In figure 1 we assume that, because of cooling of the cool, dry air by precipitation, the front in the north-south vertical cross-section, originally denoted by the line ABC, changes into the line DC. The problem is to find the displacement southward of the front, δ=AD, and also the kinetic energy liberated by the resulting decrease in potential energy.

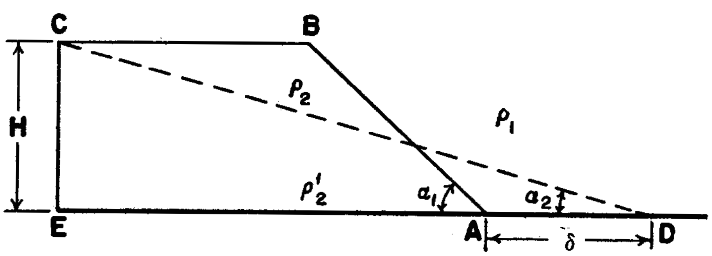

FIGURE 1.—North-South vertical cross-section showing positions of the front before adjustment (ABC) and after adjustment (CD).

The uniform density of the cool, dry air before cooling by rain is ρ_2, after cooling, ρ_2'; the density of the warm air is ρ_1, the original frontal angle, α_1, and the final angle, α_2. H is the undisturbed depth of the cool air mass.

COMPUTED AND OBSERVED DISPLACEMENTS OF FRONT

From the Margules formula for the slope of the front, where it is implicitly assumed that the frontal slope is much larger than the slope of an isobaric surface, we have

$$(1) \quad \begin{cases} \cot \alpha_1 \cong -\dfrac{g}{f}\dfrac{\rho_2-\rho_1}{\rho_2\,\Delta u_1} = \dfrac{g}{fT}\dfrac{\Delta T_1}{\Delta u_1} \\[2ex] \cot \alpha_2 \cong -\dfrac{g}{f}\dfrac{\rho'_2-\rho_1}{\rho'_2\,\Delta u_2} = \dfrac{g}{fT}\dfrac{\Delta T_2}{\Delta u_2} \end{cases}$$

where

g is the acceleration of gravity
f is the Coriolis parameter
T is a mean virtual temperature of the air masses in °K.
ΔT_1, ΔT_2 are the initial and final virtual temperature differences across the front, and
Δu_1, Δu_2 are the initial and final difference across the front of the wind components parallel to the front.

Conservation of mass of the warm air before and after adjustment leads to

$$(2) \quad \delta = \frac{H}{2}(\cot \alpha_2 - \cot \alpha_1)$$

which with aid of equation (1) reduces to

$$(3) \quad \delta = \frac{gH}{2fT}\left(\frac{\Delta T_2}{\Delta u_2} - \frac{\Delta T_1}{\Delta u_1}\right)$$

The analysis here, as in Starr's computations, goes from an initial (unstable) state to a final (stable) configuration. A study of the details of the readjustment process is not made although it is clear that an inertial oscillation of the front about its final equilibrium position will result as the front first overshoots, then is forced back, and so on, with decreasing amplitude. The period of an inertial oscillation is one-half a pendulum day, or 24 hours at latitude 30°, and it is likely that, friction neglected, the period of oscillation of the front has approximately the same value. A portion of the potential energy liberated when the frontal slope decreases as a result of precipitation cooling, must appear both as north-south oscillations of the front and as increased winds; the remainder being dissipated as a result of surface friction.

Referring now to the material in the preceding paper [2], let us see if frontal oscillations occurred and how the computed and observed values of frontal displacement agreed.

In figure 2 are presented two arrow plots showing displacements of the front along meridians 94°W. (roughly Texarkana to Shreveport, La.) and 100°W. (through Abilene, Tex.), as taken from the 3-hourly weather charts, including those shown in [2]. There are not enough stations to guarantee accuracy of these displacements, but at least they represent the best judgment of an experienced analyst taking into account all the available observations.

Both plots in figure 2 show oscillations of the front along their respective meridians. Along the 94th meridian at 0300 GMT on May 11, 1953, the front moves in an apparent damped oscillation until 2100 GMT, with a period of 6 to 9 hours and a maximum displacement of 70 km. southward. Then the front starts another series of oscillations of larger period and amplitude—18 hours and maximum displacement of 330 km. toward the north. This oscillation shows the effect of a cold front passage from the west following the movement eastward of a small wave cyclone along the front. However, before this occurred the total displacement southward from 0630 GMT on the 11th to 0630 GMT on the 12th was 260 km.

To compute frontal displacements from equation (3) we must have values of ΔT_1, ΔT_2, Δu_1, Δu_2. It is difficult to obtain representative values of these virtual temperature and wind differences across the front, especially when the frontal zone is so narrow compared to the distance between aerological stations. As nearly as can be judged from the data, there was initially no difference in *dry-bulb temperature* across the southern portion of the front—see, for example, figure 20 of [2], where at 0300 GMT, May 11, at 850 mb., dry-bulb and dew-point temperatures at Ft. Worth are 17° C., −5° C. as compared to 17° C., 12° C. at Shreveport. On the other hand, the difference in virtual temperature between the two stations is 1.3° C. This is probably larger than the difference right at the front. Accordingly, values of δ will be computed using an initial virtual temperature difference of $\Delta T_1=1°$ C.

To obtain an estimate of the final virtual temperature difference, refer to figures 8 and 16 of [2] where at 1,000 mb. (just above the surface) the dry-bulb temperature at Shreveport, La., dropped 8° C. from May 11, 0300 GMT to May 12, 0300 GMT. Most of this drop is attributed to precipitation cooling of the cool dry air. This value fits in well with an observed 6° C. average cooling in the lowest 3 km. from 0200 to 2100 GMT on May 11 at Fort Smith, Ark., and the 6° to 8° C. wet-bulb temperature depression in the same layer at 0200 on May 11 (see figure 2 of [2]). We shall therefore take 7° as the decrease in dry-bulb temperature, or 6° as the decrease in virtual temperature, induced by evaporation of precipitation falling through the cool air, thus giving $\Delta T_2=7°$ C.

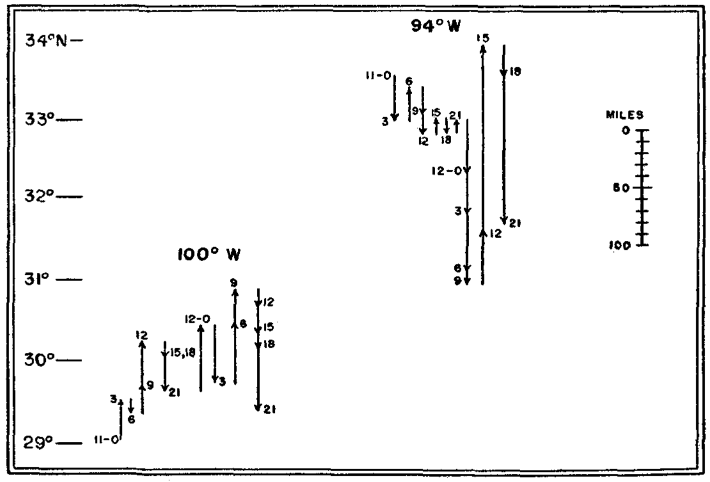

FIGURE 2.—Frontal oscillations along meridians 94° W. and 100° W. during period 0030 GMT May 11 to 2100 GMT May 12. Mileage scale applies only to distance along the meridians. Numerals at arrow-ends refer to dates and times (GMT).

Estimates of representative wind differences across the front are even more difficult to make. At the 850 mb. surface for 0300 GMT, May 11 (figure 20 of [2]) the winds at Ft. Worth and Shreveport are from the southwest, 10 knots and 40 knots, respectively, or a difference of 30 knots (15 m./sec.) across the front. In the Ft. Smith sounding, plotted in figure 8 of [2] the SSW wind increases from 20 knots below the front to 40 knots above the front, a difference of 20 knots or 10 m./sec. Thus, a value of $\Delta u_1=10$ to 15 m./sec. appears reasonable. From the Ft. Smith and Shreveport soundings 24 hours later (figure 16 of [2]) a reasonable value for Δu_2 appears to be 35 knots or 17.5 m./sec. The values of the frontal displacement in table 1 have been computed using the various values of the temperature and wind speed differences denoted above.

The observed value of 260 km. is larger than the values in the first column but smaller than the value computed for $\Delta u_1=\Delta u_2=10$ m./sec.

Turning now to the frontal motions along the 100th meridian, as depicted in figure 2, there are irregular oscillations—varying in period from 6 hours to 18 hours and in total amplitude from 90 km. to 140 km. The San Angelo, Tex., sounding (figure 3 of [2]) shows a cooling in the first kilometer of 3° to 5° from 0300 to 1500 GMT on May 11 and a spread of wet-bulb temperature depression from 10° to 17° C. In the last two columns of table 1, frontal displacements have been computed for $\Delta T_2=4°$C. first under the assumption that the frontal wind difference increases from 10 m./sec. to 17.5 m./sec., and second, that it is constant at 10 m./sec. For the first assumption $\delta=60$ km. and for the second, $\delta=140$ km.; the observed displacements are between these values.

TABLE 1.—*Computed frontal displacements ($H=2$ km., $T=290°$ K., $f=0.73\cdot 10^{-4}$ sec.$^{-1}$ (30° lat.)*

ΔT_1 (° C.)	1	1	1	1	1	1
ΔT_2	7	7	7	7	4	4
Δu_1 (m./sec.)	10	15	10	15	10	10
Δu_2	17.5	17.5	10	15	17.5	10
δ (km.)	140	155	280	185	60	140

Thus, in summary, it may be said that if one interprets the oscillations of the fronts as precipitation-induced, then there is reasonably good agreement between computed and observed maximum displacements; the character and period of the frontal motions suggest inertial oscillations, sometimes damped by friction, and other times reintensified apparently by new surges of the rain-cooled air north of the front.

LIBERATION OF KINETIC ENERGY

To obtain an estimate of the kinetic energy liberated by the adjustment of the front, let us now compute the difference ΔE, between initial and final potential energy of the mass distribution shown in figure 1 *after the cooling has taken place.* Referring to figure 1 and making use of equation (2), we find for a 1-cm. thick vertical slab of air

$$\Delta E=\Delta E_c+\Delta E_w=\frac{gH^2}{6}(\rho'_2-\rho_1)\delta \tag{4}$$

Neglecting friction,

$$\Delta E=\frac{1}{2}M(\overline{V}^2-\overline{V}_0^2)=\frac{H^2}{4}\cot\alpha_2(\rho'_2+\rho_1)(\overline{V}^2-\overline{V}_0^2) \tag{5}$$

where M is the total mass of the 1-cm. thick air slab above line DE and $\overline{V}$, $\overline{V}_0$ are final and initial average wind speeds in the same mass.

Equating (4) and (5), and making use of equations (1) and (2),

$$\overline{V}^2-\overline{V}_0^2=\frac{gH}{3}\frac{\rho'_2-\rho_1}{\rho'_2+\rho_1}\left[1-\left(\frac{\rho_2-\rho_1}{\rho'_2-\rho_1}\right)\frac{\rho'_2\,\Delta u_2}{\rho_2\,\Delta u_1}\right]$$

$$=\frac{gH}{3}\frac{T_1-T'_2}{T_1+T'_2}\left[1-\left(\frac{T_1-T_2}{T_1-T'_2}\right)\frac{\Delta u_2}{\Delta u_1}\right]$$

Taking $H=2\cdot10^5$ cm., $\Delta u_2=\Delta u_1$, $T_1-T'_2=8°$, $T_1-T_2=1°$, $\frac{T_1+T'_2}{2}=290°K.$, and $\overline{V}_0=10$ m./sec.$=20$ knots, we find $\overline{V}=13.4$ m./sec.$=27$ knots.

Thus, 3.4 m./sec. or 7 knots is the average increase in wind speed in the system caused by flattening out of the front by precipitation cooling. This is an overestimate since friction is neglected. If the liberated kinetic energy is not spread over the entire system of air masses but is localized in certain areas this would mean high local winds, such as occurred in portions of the cool air mass near the front.

REFERENCES

1. V. P. Starr, "The Readjustment of Certain Unstable Atmospheric Systems under Conservation of Vorticity," *Monthly Weather Review,* vol. 67, no. 5, May 1939, pp. 125–134.
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